Investigating dynamical systems using Optic-Fluidics

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Abstract

We are presenting experimental results and simulations of dynamical systems using magneto-optics. These light patterns are obtained by the observation of a thin film of ferrofluid in the presence of a magnetic in the presence of a magnetic field.

1. Introduction

In our previous work, we have considered the analogy between the general properties of vector fields of the phase space of dynamical systems with the properties of potential of magnetic charges using magneto-optics [1], as it is shown in Fig. 1.

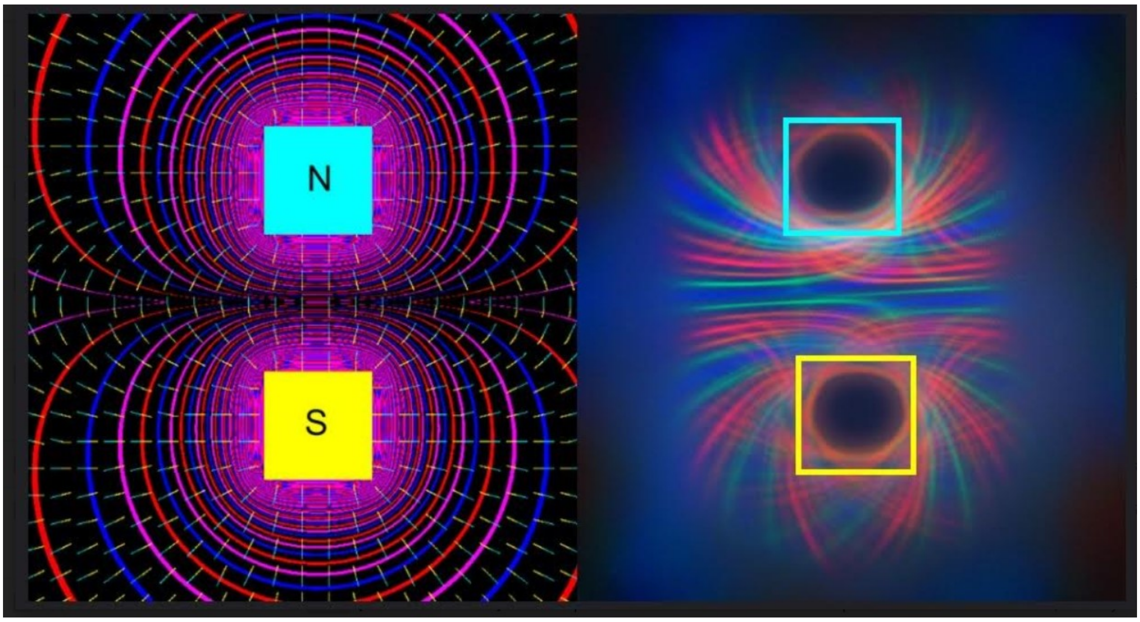

Figure 1 – (a) Two magnetic charges with the field lines represented by the arrows and isopotentials represented by the colored lines and its magneto-optical counterpart in (b).

We have proposed this representation because the representation of both fields is comparable, the existence of two different types of “charges” enable us to obtain elliptic points and saddles. Basically, the colored lines observed from our magneto-optical system are obtained from the light diffraction of light sources in microneedles aligned with magnetic field. In Fig. 1 and Fig.2, we can see that the isopotentials are perpendicular to the lines of the magnetic field.

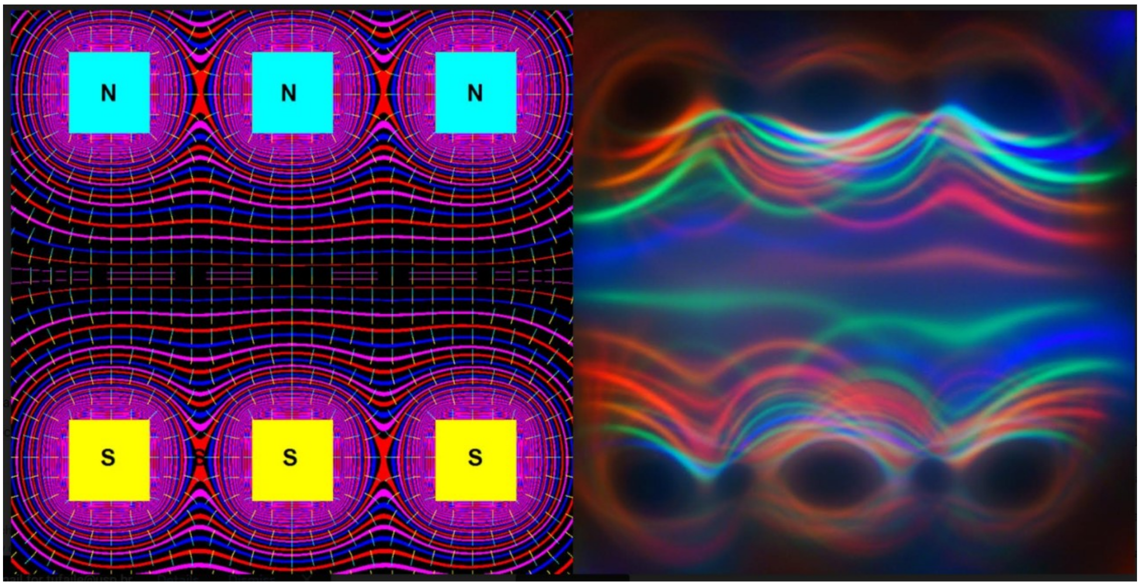

Figure 2. – (a) Three magnetic charges with the field lines represented by the arrows and isopotentials represented by the colored lines and its magneto-optical counterpart in (b).

It is important to note that the representation of magnetic charges is a valid way to represent the magnetic field and this is not incompatible with the idea of the Lorentz force, as the same way that a phase space represents states of motion, not the motion itself.

However, there is an apparent contradiction in this analogy, because the representation of isopotentials and the colored lines of our magneto–optical is not perfect. A close observation of the light patterns of the experiment shows the existence of crossing lines, which could imply in indeterminacy in a dynamical system, violating the classical representation of dynamical systems. The light patterns mimic the isopotentials, because the light patterns are a combination of the magnetic field and the position of the light source. For different positions of the light source, we have different diffracted lines, which eventually will cross each other. In this way, metaphorically speaking, these luminous patterns linked to the isopotentials are equivalent to the representation of the nature by the impressionist painters, with emphasis in depiction of light in its changing qualities with unusual visual angles.

Consider now the Henon conservative map given by:

$$\begin{aligned} x' &= x \cos\alpha - (y - x^2)\sin\alpha\,; \\ y' &= x \sin\alpha + (y - x^2)\cos\alpha. \end{aligned} \qquad (1)$$

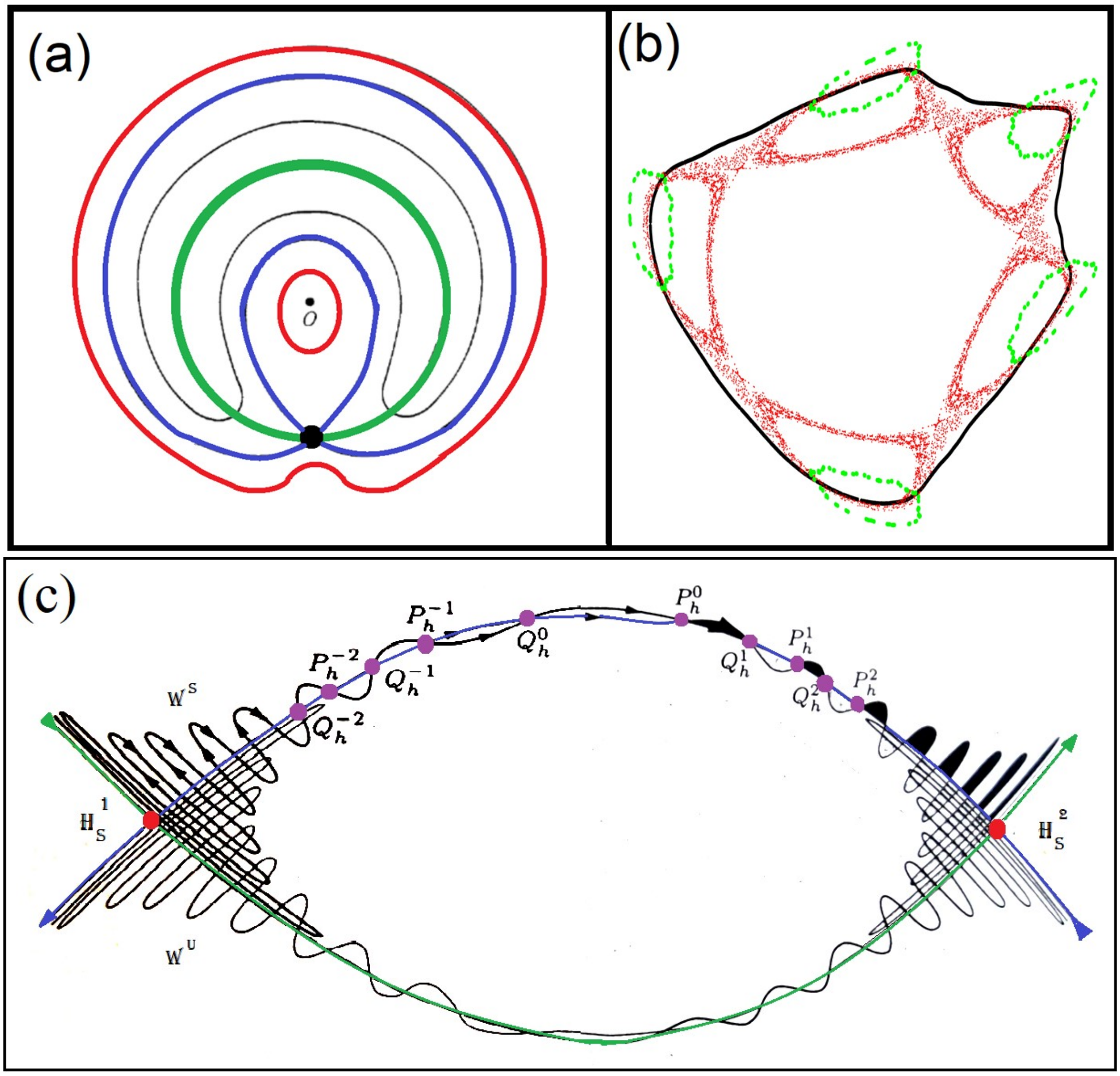

Figure 1.3 – In (a) diagram of separatrix chaos. In (b) the diagram of a Henon-Heyles map with chaotic oscillations. In (c) the concept of chaos in conservative systems close of hyperbolic points.

For the case of Hamiltonian systems, we can observe the existence of chaos for perturbations close to the separatrix of the system, as it is shown in Fig. 1.3(a), which shows the conditions of the nonlinear resonance on the phase space, in which the green line is the unperturbed trajectory. The blue line is the new separatrix of the phase oscillations. The classical plot of chaotic behavior can be obtained for the Henon conservative map of the Fig. 1.3(b) from eq. (1), with chaotic behavior given by red region ($\alpha = \pi/2$-0.228), quasiperiodic behavior in green color ($\alpha = \pi/2$-0.200), and another chaotic region in black ($\alpha = \pi/2$-0.250).

The idea o f chaos in this case can be understood if we follow the stable (W^s) and unstable manifolds (W^u) of Fig. 1.3(c), until their intersection points in red, called homoclinic points H_S^1 and H_S^2. Applying the perturbation repeatedly to P_h^0, we have the sequence of image points P_h^k converging towards hyperbolic point for k tending to infinite, and consequently W^u and W^s can only intersect after an infinite sequence, and the same is valid for the reversing points Q_h^k. The result is an extraordinary complex view of intersecting invariant manifolds. One example of this behavior is shown in Fig. 1.4 using the eq. (1), for the case of α equals to ($\pi/2$-0.228). (see ref. 2).

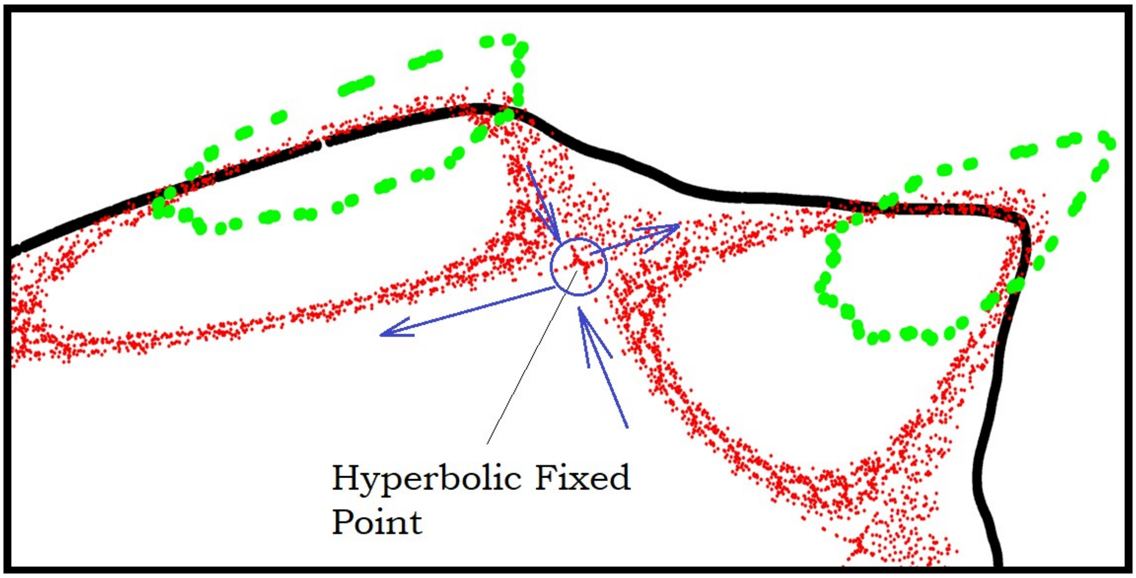

Figure 1.4 – Chaos close of a hyperbolic point in the Hénon map.

For the case of the magneto-optics in our experiment, we observed that the light patterns is oriented by the vectorial product of Fig. 1.5 [1, 3, 4, 6, 7].

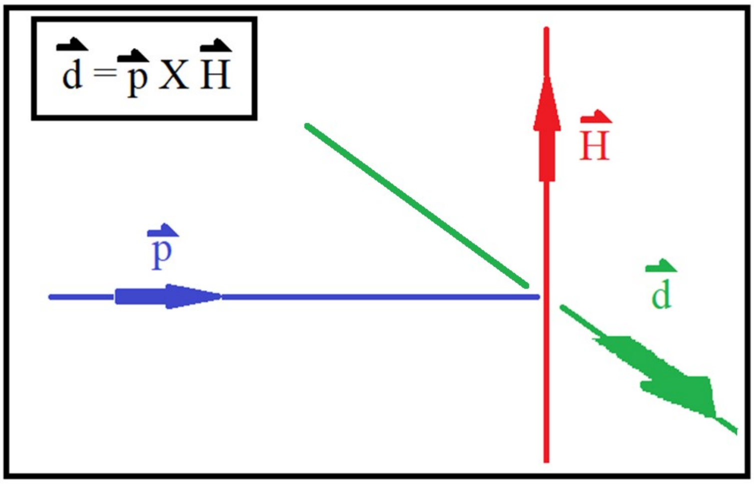

Figure 1.5 - The vector **d** is the direction of the tangent line of the diffracted lines, which is perpendicular to the direction of propagation of the light ray **p** and the orientation of the magnetic field **H**.

This approach is well known in celestial mechanics, demonstration of chaotic pendulum, or in electromagnetism, where physicists look for the dynamics of particles in magnetic fields. We are investigating in this paper the equivalent case of chaotic scenario in magnetostatics interacting with light from our experiment involving magneto-optics. We were inspired by the direct observation of luminous patterns and properties of magnetic fields.

2. Experimental Apparatus and Modeling Isopotentials

In Fig.2.1 we present the experimental apparatus of this system. The luminous patterns observed in the thin film of ferrofluid is a direct effect of the magnetic field with the iron particles, which take a shape that scatters light in a certain shape for the viewer. In this way we have to use an array of magnets of Fig. 2.2, above the magnets we have a mirror. The device known as Ferrolens of Fig. 2.3, the Hele-Shaw cell containing the ferrofluid, is placed above this assembly. We use different light arrays above this setup, which represent the ferro-mirror experiment. The ferrofluid is a stable colloidal dispersion using light mineral oil. The nanoparticles are spheres of the order of 10nm in diameter. The magneto-optic effect results in the change of some optical parameters of the ferrofluid, forming images. For more details see Refs 1, 3, 5 and 6.

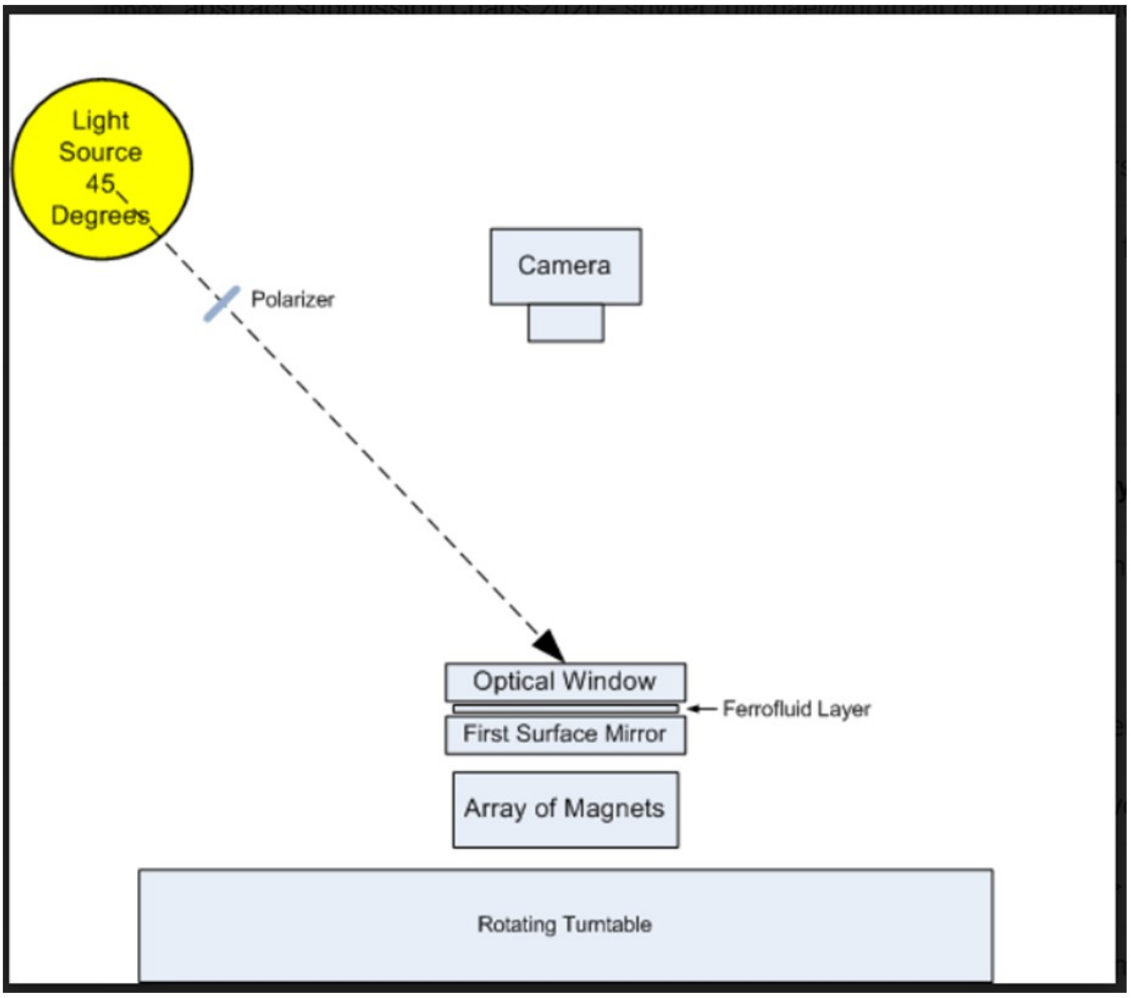

Figure 2.1 Diagram of the ferro-mirror experiment setup.

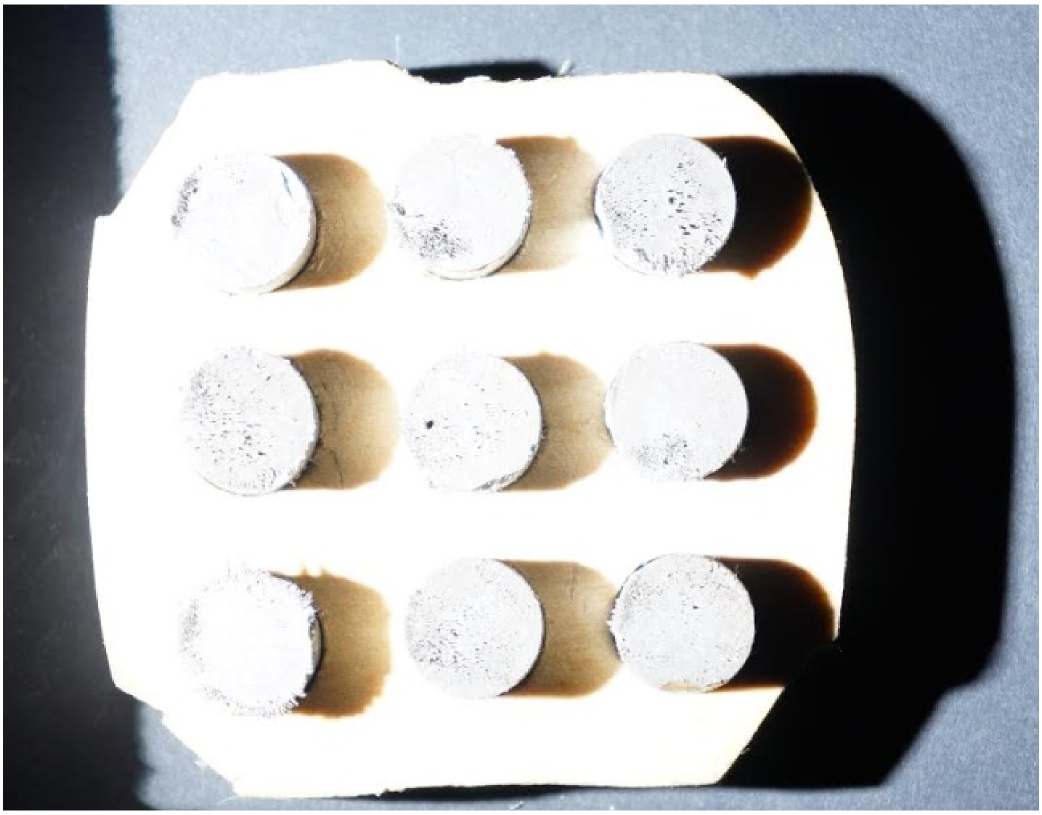

Figure 2.2 - Magnets fixed in a base.

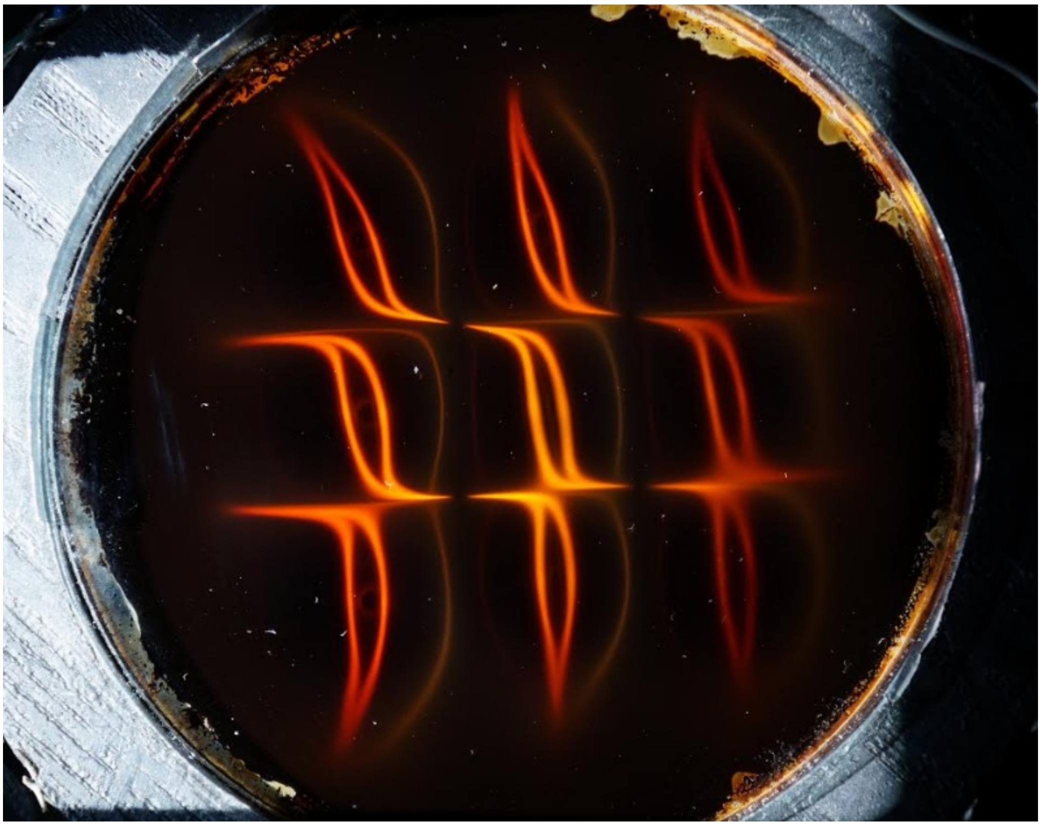

Figure 2.3 – Pattern obtaine with the Ferrolens.

The software Pic2Mag [5] simulates some aspects of magnetic field arrangement of a magnet array, such as vector field and isopotentials, like the case of two magnets of Fig. 2.4(a), wich can be compared to a phase space like the one in Fig. 2.4(b), which represents the phase space of a pendulum.

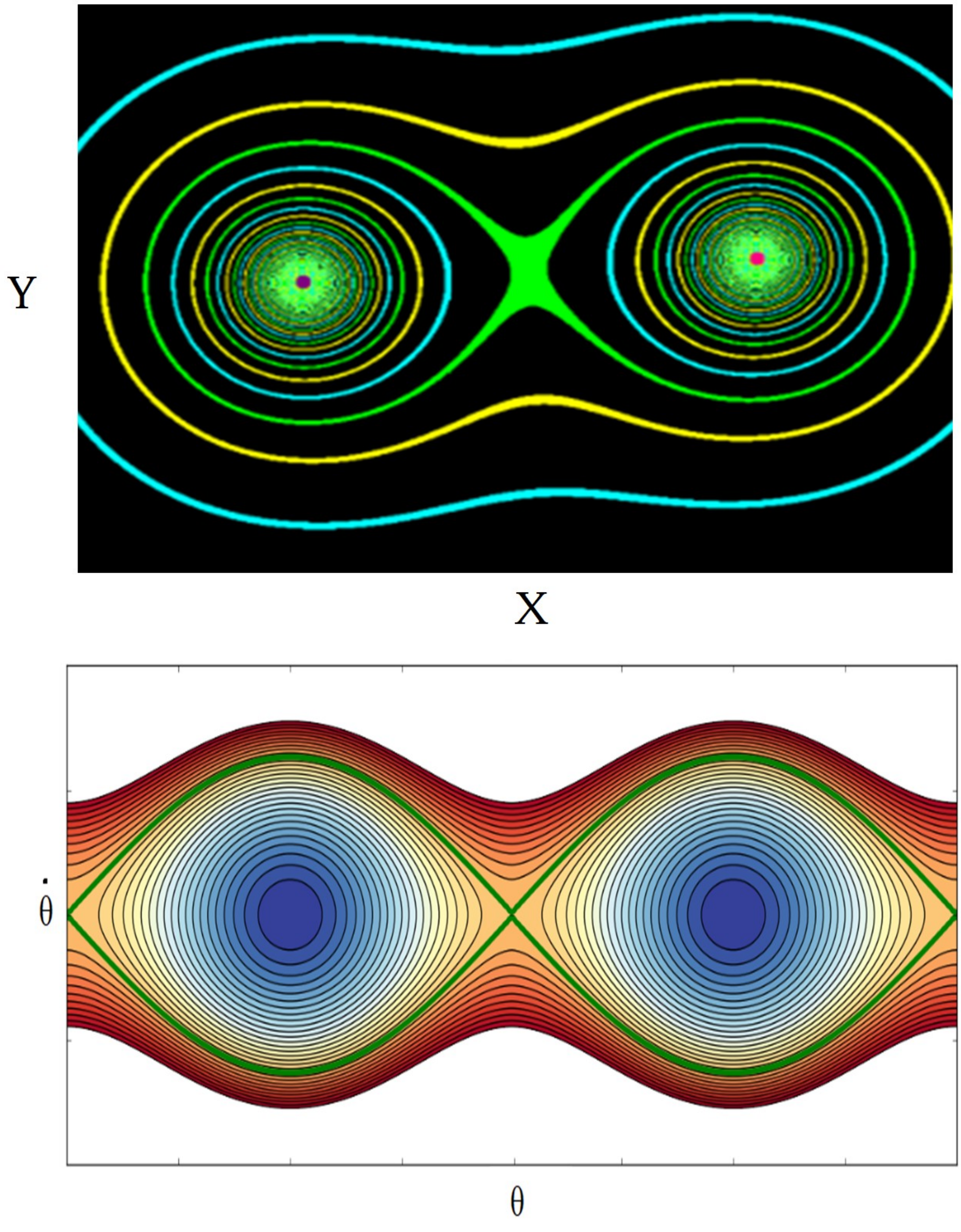

Figure 2.4 – Comparison between isopotentials of magnetic field and the phase space of a pendulum. The green curves represents the separatrix.

3. The patterns of eyes and Chirality.

One interesting phenomenon observed is a meagnetochiral pattern of Fig. 3.1, which there is three magnets to create this pattern, in a tripolar configuration formed by south-north-south poles. The pattern resembles three eyes arranged in a column-like alignment. Fig. 3.1(c) is the top view of the light pattern, Figs. 3.1 (a)-(b) is from the same system observed from the right side, and Figs. 3.1(d)-(e) are perspective obtained from the left. We can see that the pattern suffers distortions. However, these patterns cannot be overlapped with other in order to be reproduced. One image is a reflection of the other, in a such way that there is chiral effect. We can consider that the assembly of nanoparticles is somehow affecting these light patterns, because nanoscale particles could self-assemble into helical-like structures due to the interplay of magnetic dipoles and van der Walls interactions [3, 4, 7]. The consequence of this anisotropy is the emergence of optical chiral structures.

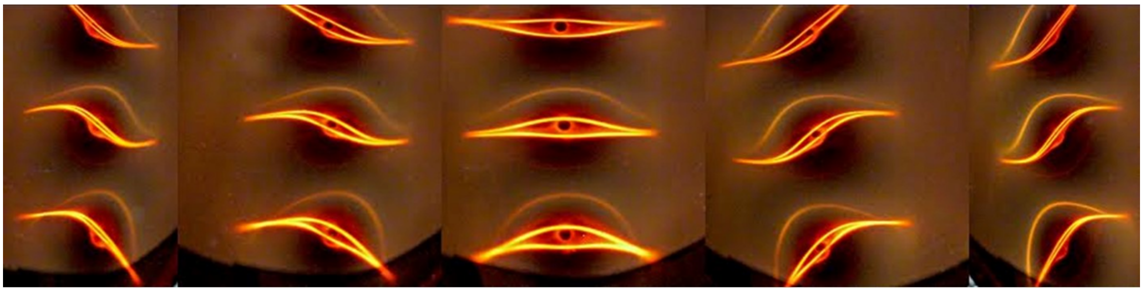
Figure 3.1- Magneto chiral effect.

4. Hyperbolic points

For the case of Hamiltonian systems, the existence of saddle point is the important key to observe the existence of chaos. The stable and unstable manifolds are called separatrices, and when a weak perturbation is added, the separatrix are destroyed and replaced by a separatrix chaotic layer. The same way as the separatrix is obtained numerically by integration of the equations with a set of initial conditions in the vicinity of the separatrix, we can explore in our system what is happening around the saddle points in our experiment. Let’s consider othe case of Fig. 4.1 with a configuration of isopotentials equivalent of a torus. Zooming the central area of this image in Fig. 4.2(a) with the experiment in Fig. 4.2(b), we can see what is happening with the four saddle points around the center of the light pattern observed with experiment. The colored lines converge to the saddle point and vanish. In contrast, the center point of the image, which represents the center point of a dynamical system, the colored lines swirls around it, and the central region is dark.

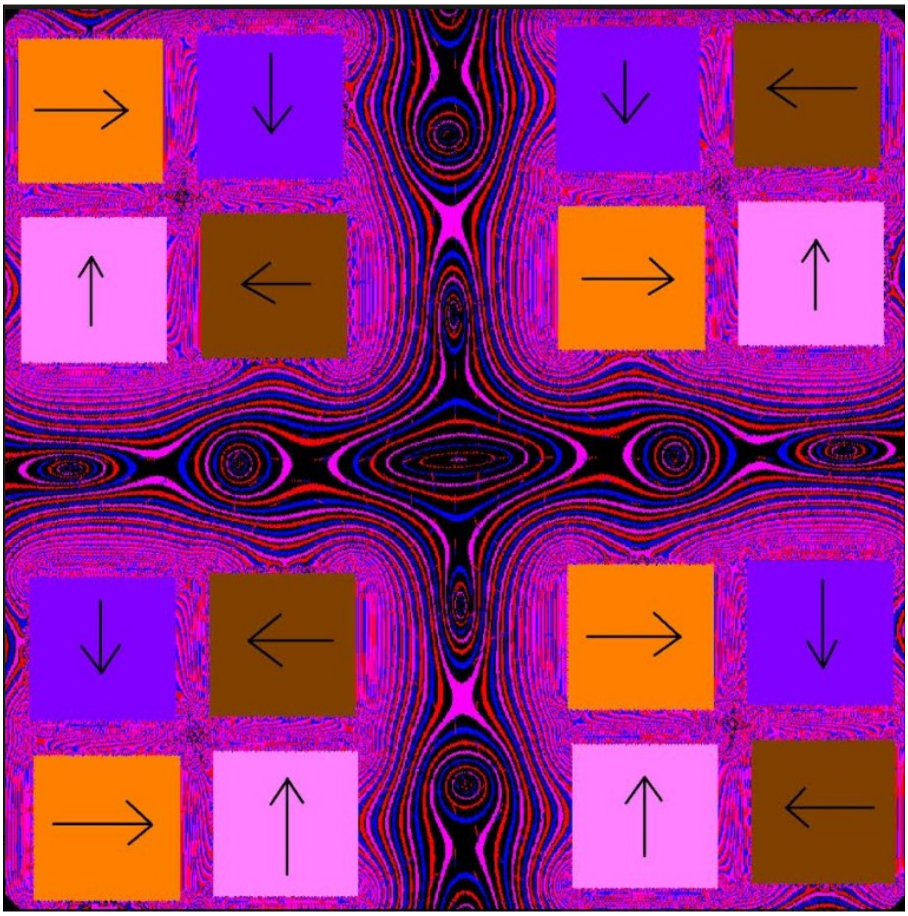

Figure 4.1 Simulation of isopotentials in a a torus.

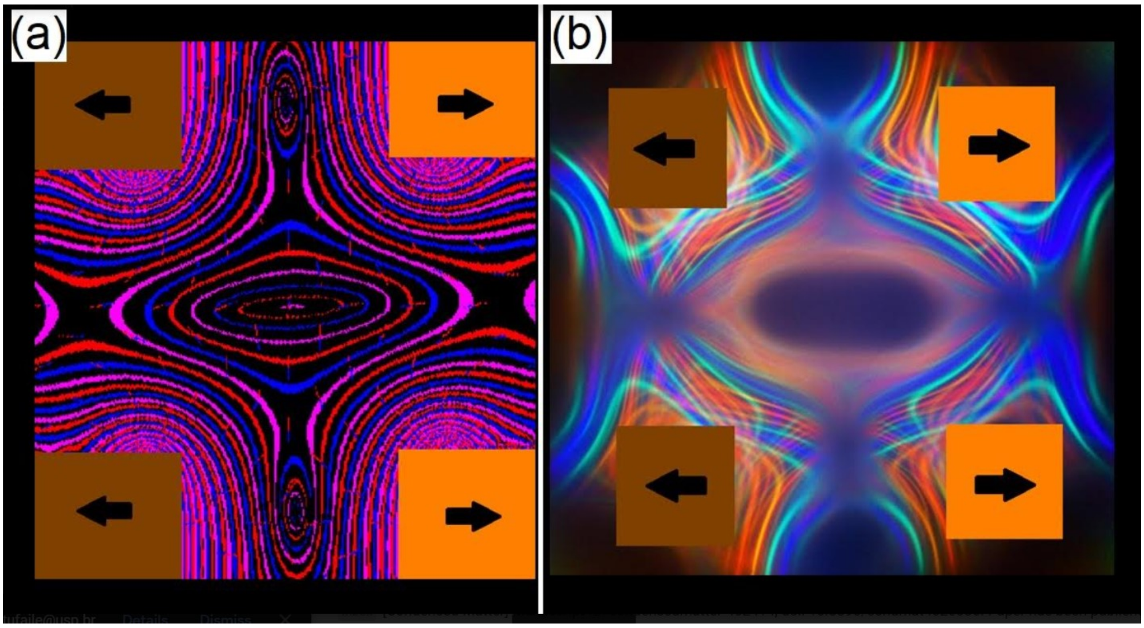

Figure 4.2 Exploring Center and saddle points: simulation and experiment

Fig. 4.3 was obtained by placing the pattern obtained experimentally on the simulation. With this picture, we can observe that the saddle points of the simulation is slightly different from the experiment, for example the green cross at the right side, at the top of Fig. 4.3.

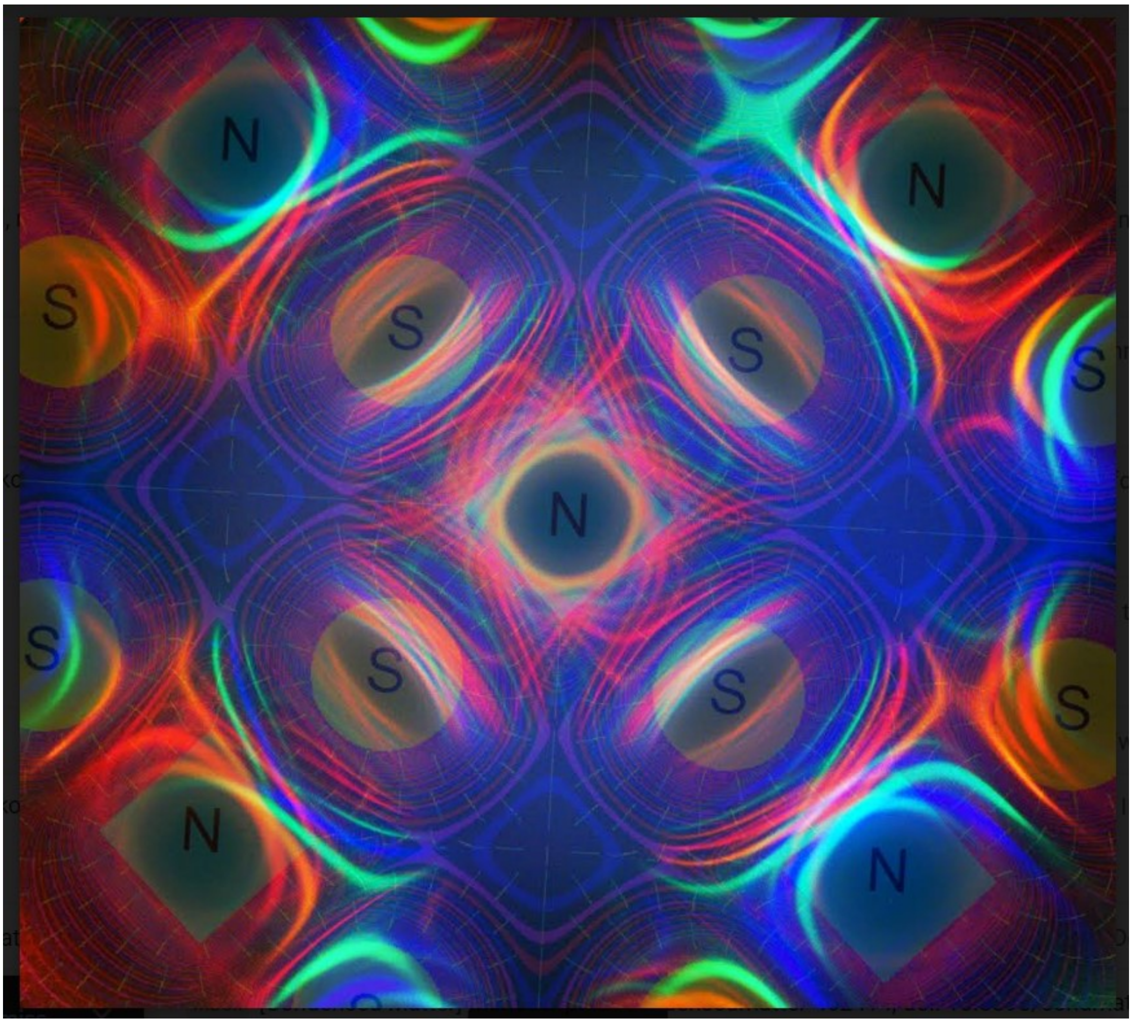

Figure 4.3 –Another array of magnets superposed on the simulation of the magnetic field and isopotentials.

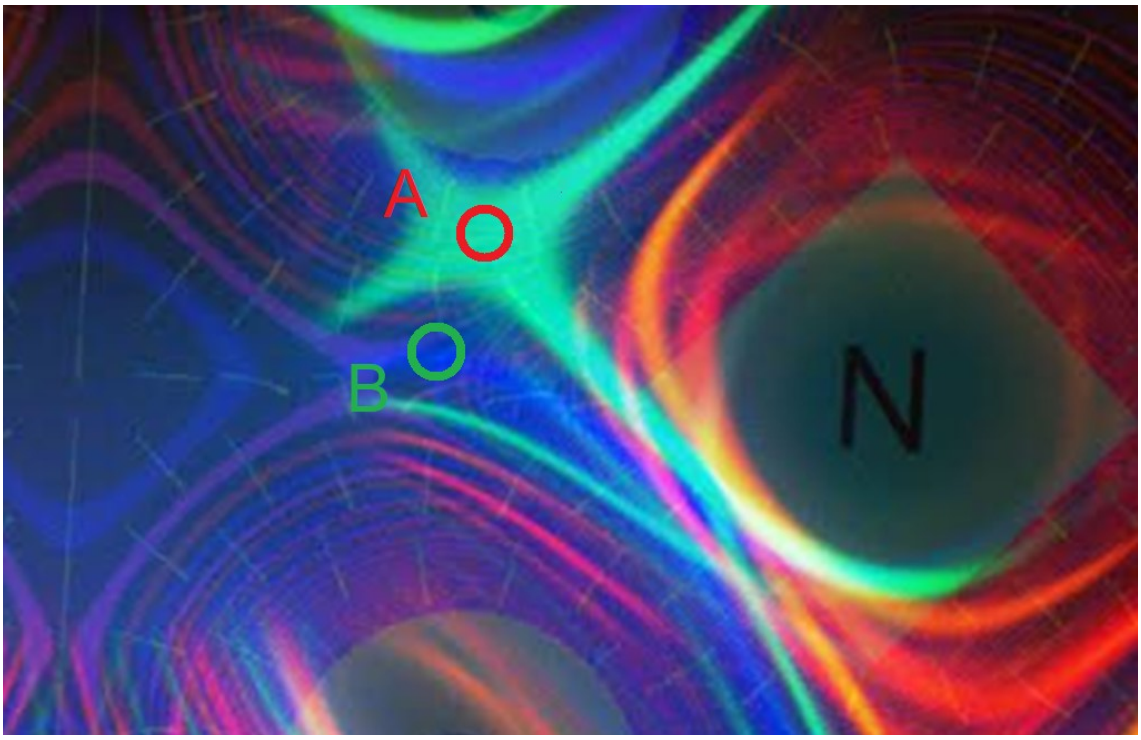

Figure 4.4 Observing a hyperbolic magnetic point from the previous figure: the experiment is the red circle A, and simulation is the green circle B.

Conclusions

We have explored some aspects of the analogy between dynamical systems and the magneto-optical system formed by a thin film of ferrofluid. Magnetic static fields have some general properties of Hamiltonian systems, and using different magnetic fields configurations, we look for hyperbolic points and observed how the experiment behaves around these regions. The light patterns are related to the vectorial product between the ray light **p** and the orientation of the magnetic field **H**, given locally the tangent vector **d**. During theses explorations of this magneto-optical system, we have found some evidences of chiral effects and we suggested that this effect is related to anysitropic properties of magnetic nanoparticles.

We have observed that the presence of the thin film of ferrofluid affects the magnetic field, and the formation of patterns can show the differences between the values of the patterns observed experimentally and the computed values.

Acknowledgements

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5) Program for simulation of magnetic field: www.pic2mag.com

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